

# Hawking Radiation of Fermionic Field and Anomaly in 2+1 Dimensional Black Holes

Ran Li,<sup>\*</sup> Shi Li,<sup>†</sup> and Ji-Rong Ren<sup>‡</sup>

*Institute of Theoretical Physics, Lanzhou University, Lanzhou, 730000, Gansu, China*

## Abstract

The method of anomaly cancellation to derive Hawking radiation initiated by Robinson and Wilczek is applied to 2+1 dimensional stationary black holes. Using the dimensional reduction technique, we find that the near-horizon physics for the fermionic field in the background of the general 2+1 dimensional stationary black hole can be approximated by an infinite collection of two component fermionic fields in 1+1 dimensional spacetime background coupled with dilaton field and  $U(1)$  gauge field. By restoring the gauge invariance and the general coordinate covariance for the reduced two dimensional theory, Hawking flux and temperature of black hole are obtained. We apply this method to two types of black holes in three dimensional spacetime, which are BTZ black hole in Einstein gravity and a rotating black hole in Bergshoeff-Hohm-Townsend (BHT) massive gravity.

PACS numbers:

Keywords:

---

<sup>\*</sup>Electronic mail: liran05@lzu.cn

<sup>†</sup>Electronic mail: lsh08@lzu.cn

<sup>‡</sup>Corresponding author. Electronic mail: renjr@lzu.edu.cn

## I. INTRODUCTION

Using the techniques of quantum field theory in curved spacetime, Hawking made the discovery[1, 2] that black hole can radiate from its event horizon with a purely thermal spectrum at the temperature  $T = \frac{\kappa}{2\pi}$  ( $\kappa$  is the surface gravity of the black hole), which leads to a remarkable connection between thermodynamics, quantum mechanics and gravity [3]. Due to the significance of Hawking radiation and the difficulties existing in applying the Hawking's original method to more complicated spacetime backgrounds, several derivations of Hawking radiation have been proposed in literatures, including the so-called Damour-Ruffini method[4], trace anomaly method [5], quantum tunneling method[6] and gravitational anomaly method[7, 8].

The recent proposal of deriving Hawking radiation via gravitational and gauge anomalies proposed by Wilczek and his collaborators [7, 8] has attracted a lot of interests. This rejuvenates the interest of investigation of Hawking radiation. For various types of black holes, these investigations have been carried out [9]. In fact, the anomaly analysis can be traced back to Christensen and Fulling's early work [5], in which they suggested that there exists a relation between the Hawking radiation and anomalous trace of the field under the condition that the covariant conservation law is valid. Imposing boundary condition near the horizon, Wilczek et al showed that the Hawking radiation is just the cancel term of the gravitational anomaly of the covariant conservation law and gauge invariance. Their basic idea is that, near the horizon, a quantum field in a black hole background can be effectively described by an infinite collection of (1+1)-dimensional fields on  $(t, r)$  space, where  $r$  is the radial direction. One could treat the original higher dimensional field as a collection of two-dimensional quantum fields using the dimensional reduction. In this two-dimensional reduction, because all the ingoing modes can not classically affect physics outside the horizon, the two-dimensional effective action in the exterior region becomes anomalous with respect to gauge or general coordinate symmetries. To cancel the anomaly, they found the Hawking flux is universally determined only by the value of anomalies at the horizon. It is also interesting to generalize the above idea to derive the high-spin flux from a black hole through the conformal symmetry in reduced  $(1+1)$ -dimensional background[10]. There are also some interests to investigate the relationship between quantum anomaly and tunneling method[11].

Originally, in order to apply the anomaly cancellation method, only scalar field is considered in the dimensional reduction process. However, the anomaly formalism can also be used to investigate the Hawking radiation of vector field[12] and spinor field[13]. In [12], it is explicitly shown that the theory of an electromagnetic field on  $d$ -dimensional spherical black holes can be reduced to one of an infinite number of massive complex scalar fields on 2-dimensional spacetime, for which the usual anomaly cancellation method is available. For the Hawking radiation of spinor field[13], it turns out that the near-horizon physics for fermionic field in Kerr black hole can be approximated by an effective two dimensional field theory. Therefore, in order to verify the universality of the anomaly cancellation method in deriving Hawking radiation, it is interesting to discuss the Hawking radiation for other fields with nonzero spins.

In the present work, we extend the gravitational anomaly method to investigate the Hawking radiation of fermionic field in the 2+1 dimensional stationary black hole. Using the dimensional reduction technique, we find that the near-horizon physics for the fermionic field in the background of the general 2+1 dimensional stationary black hole can be approximated by an infinite collection of two component fermionic fields in 1+1 dimensional spacetime background coupled with the dilaton field and the  $U(1)$  gauge field. By restoring the gauge invariance and the general coordinate covariance for the reduced two dimensional theory, the Hawking temperature and flux of black hole are obtained. We apply this method to two types of black holes in three dimensional spacetime, which are BTZ black hole[15] in Einstein gravity and a rotating black hole[18] in Bergshoeff-Hohm-Townsend (BHT) massive gravity[17]. It turns out that the Hawking temperatures are recovered and they are consistent with the results obtained previously in literatures.

This paper is organized as follows: In section 3, by performing the dimensional reduction for the fermionic field action in the general 2+1 dimensional stationary black hole background, we obtain the reduced 1 + 1 dimensional effective theory and calculate the Hawking flux via anomaly equations near the horizon. In section 3 and 4, the method is applied to BTZ black hole in Einstein gravity and a rotating black hole in BHT massive gravity. The last section is devoted to conclusions and discussions.

## II. HAWKING RADIATION OF FERMIONIC FIELD AND ANOMALIES

In this section, let us consider the general 2+1 dimensional stationary black hole space-time, of which the line element is given by[14]

$$ds^2 = -h(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\phi + N^\phi(r)dt)^2 . \quad (1)$$

We assume that the event horizon is located at  $r = r_H$ . Meanwhile the metric functions satisfy the equations  $h(r_H) = 0$  and  $f(r_H) = 0$ . The angular velocity of the black hole is given by  $\Omega_H = -N^\phi(r_H)$ . The surface gravity can be calculated from the standard formula  $\kappa^2 = -\frac{1}{2}k_{\mu;\nu}k^{\mu;\nu}|_{r_H}$ , where the killing vector  $k$  is given by

$$k = \partial_t + \Omega_H \partial_\phi . \quad (2)$$

After some algebra, one can obtain

$$\kappa = \frac{1}{2} \sqrt{\frac{f}{h}} h' \Big|_{r=r_H} , \quad (3)$$

where the prime ' denotes the derivative with respect to the coordinate  $r$ . For simplicity, we will consider the special case when  $h(r) = f(r)g(r)$  and  $g(r_H) \neq 0$ . In this case, the surface gravity can be rewritten as  $\kappa = \frac{1}{2} \sqrt{f'(r_H)h'(r_H)}$ . The Hawking temperature given by the surface gravity is  $T_H = \frac{\kappa}{2\pi} = \frac{1}{4\pi} \sqrt{f'(r_H)h'(r_H)}$ .

We consider the action for the fermionic field in this background

$$\begin{aligned} S &= \int d^3x \sqrt{-g} \bar{\psi} \gamma^a e_a^\mu D_\mu \psi \\ &= \int d^3x \sqrt{-g} \psi^\dagger \gamma^0 \gamma^a e_a^\mu \left( \partial_\mu - \frac{1}{8} \omega_{bc\mu} [\gamma^b, \gamma^c] \right) \psi , \end{aligned} \quad (4)$$

where the spin connection is given by  $\omega_{ab\mu} = e_{a\nu} \nabla_\mu e_b^\nu$ . It should be noted that the indices  $a, b, c = 0, 1, 2$  are flat while the indices  $\mu, \nu = t, r, \phi$  are curved.  $\gamma^a$  is the gamma matrix in three dimensional flat spacetime. According to the line element (1), the tetrad field  $e_a^\mu$  can be selected as

$$\begin{aligned} e_0^\mu &= \left( \frac{1}{\sqrt{h}}, 0, -\frac{N^\phi}{\sqrt{h}} \right) , \\ e_1^\mu &= (0, \sqrt{f}, 0) , \\ e_2^\mu &= (0, 0, \frac{1}{r}) . \end{aligned} \quad (5)$$

One can also define the spin connection with three flat indices as  $\omega^a_{bc} = e^\mu_c \omega^a_{b\mu} = e^\mu_c e^a_\nu \nabla_\mu e^\nu_b$ . After some algebra, one can obtain the non-vanishing components of spin connection as listed in the following

$$\begin{aligned}\omega^1_{00} &= \frac{\sqrt{f}h'}{2h}, \\ \omega^1_{02} &= \omega^2_{01} = \omega^2_{10} = -\frac{r\sqrt{f}}{2\sqrt{h}} \frac{dN^\phi}{dr}, \\ \omega^2_{12} &= \frac{\sqrt{f}}{r}.\end{aligned}\tag{6}$$

When substituting the tetrad field and spin connection into the fermionic field action, one can obtain

$$S = \int dt dr d\phi r \sqrt{\frac{h}{f}} \psi^\dagger \gamma^0 \left\{ \frac{\gamma^0}{\sqrt{h}} (\partial_t - N^\phi \partial_\phi) + \gamma^1 \sqrt{f} \partial_r + \frac{\gamma^2}{r} \partial_\phi - \frac{1}{4} \left( \frac{\sqrt{f}h'}{h} \gamma^1 + \frac{2\sqrt{f}}{r} \gamma^1 - \frac{r\sqrt{f}}{\sqrt{h}} \frac{dN^\phi}{dr} \right) \right\} \psi.\tag{7}$$

To proceed, one can define the tortoise coordinate as

$$\frac{dr^*}{dr} = \frac{1}{f(r)}.\tag{8}$$

When taking the near horizon limit  $r \rightarrow r_H$ , the metric function  $f(r) \rightarrow 0$  and  $h(r) \rightarrow 0$ . After ignoring the sub-leading contribution of the terms in the action, the action can be effectively simplified as

$$S = \int dt dr d\phi r \sqrt{\frac{h}{f}} \psi^\dagger \gamma^0 \left[ \frac{\gamma^0}{\sqrt{h}} (\partial_t - N^\phi \partial_\phi) + \frac{\gamma^1}{\sqrt{f}} (\partial_{r^*} - \frac{fh'}{4h}) \right] \psi.\tag{9}$$

In order to perform the integral over the angular  $\phi$  in the action, one can expand the two component spinor field  $\psi$  in the following way  $\psi(t, r, \phi) = \sum_m \psi_m(t, r) e^{im\phi}$ . Note that  $\psi_m(t, r)$  is also a two component spinor field. Substituting the partial wave decomposition into the action and integrating over the angular  $\phi$ , one can obtain

$$S = \sum_m \int dt dr \frac{r}{2\pi} \sqrt{\frac{h}{f}} \psi_m^\dagger(t, r) \gamma^0 \left[ \frac{\gamma^0}{\sqrt{h}} (\partial_t - imN^\phi) + \frac{\gamma^1}{\sqrt{f}} (\partial_{r^*} - \frac{fh'}{4h}) \right] \psi_m(t, r).\tag{10}$$

Now our task is to interpret the reduced action (10) in terms of the two dimensional quantities. We consider the two dimensional metric

$$ds^2 = -h(r) dt^2 + \frac{dr^2}{f(r)}.\tag{11}$$

In this background, we have calculated the spinor covariant derivative  $D_\alpha^{(2)}$  of a two dimensional spinor field  $\chi(t, r)$  contracted with two dimensional covariant gamma matrices  $\sigma^i e_i^{(2)\alpha}$ , which is explicitly given by

$$\sigma^i e_i^{(2)\alpha} D_\alpha^{(2)} \chi = \left[ \frac{\sigma^0}{\sqrt{h}} \partial_t + \frac{\sigma^1}{\sqrt{f}} \left( \partial_{r^*} - \frac{fh'}{4h} \right) \right] \chi. \quad (12)$$

Now, we choose the following gamma matrices in three dimensional flat spacetime

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (13)$$

While the gamma matrices in two dimensional flat spacetime are selected to be  $\sigma^0 = \gamma^0$  and  $\sigma^1 = \gamma^1$ . We can rewrite the reduced action (10) in terms of the two dimensional quantities

$$S = \sum_m \int dt dr \sqrt{-g^{(2)}} \Phi \bar{\psi}^{(2)} \sigma^i e_i^{(2)\alpha} (D_\alpha^{(2)} - im A_\alpha) \psi_m^{(2)}, \quad (14)$$

where the dilaton field  $\Phi$  and gauge field  $A_\alpha$  associated with the charge  $m$  are given by

$$\Phi = \frac{r}{2\pi}, \quad A_t = N^\phi, \quad A_r = 0. \quad (15)$$

It should be noted that the tetrad fields  $e_a^\mu$  we choose are convenient in the dimensional reduction process, but not the only one. One can get a new type of tetrad fields by performing a gauge rotation for the tetrad fields presented in Eq.(5), and use the new tetrad fields in calculations. If the corresponding two dimensional tetrad fields are properly selected, the form of the reduced two dimensional metric can be preserved.

Up to now, using the dimensional reduction technique, we have found that the near-horizon physics for the fermionic field in the background of the general 2+1 dimensional stationary black hole can be approximated by an infinite collection of two component fermionic fields in 1+1 dimensional spacetime background coupled with the dilaton field and the  $U(1)$  gauge field. Therefore one can treat the original 2+1 dimensional theory as a collection of 1 + 1 dimensional quantum fields.

Now the usual anomaly cancellation method can be applied to derive the Hawking radiation of spinor field. Since the event horizon is a null hypersurface, all ingoing modes at the horizon can not classically affect physics outside the horizon. Here, we focus on the effective theory outside the horizon and ignore the the classically irrelevant ingoing modes. Then, the theory becomes chiral and gauge or gravitational anomalies arise. If the symmetries of the theory are restored, these anomalies should be cancelled by the quantum

effects of the classically irrelevant ingoing modes. It has been shown that the condition for anomaly cancellation can give rise to the Hawking flux of the charge, angular momentum and energy-momentum.

In the present case, the original rotating symmetry in 2+1 dimensional spacetime is reduced to the gauge symmetry in two dimensional background. In the region near the horizon, the covariant current equations modified by Abelian anomaly is given by

$$\nabla_\mu \tilde{J}^\mu = -\frac{m^2}{4\pi\sqrt{-g^{(2)}}}\epsilon^{\mu\nu}F_{\mu\nu} \quad (16)$$

By solving this equation with appropriate boundary conditions that the covariant gauge currents vanish at the horizon, the flux of angular momentum from the horizon is given by

$$F_a = -\frac{m^2}{2\pi}A_t(r_H) = \frac{m^2}{2\pi}\Omega_H \quad (17)$$

This is consistent with the flux derived from the Hawking distribution given by the Planck distribution with chemical potentials for angular momentums  $m$  of the fields radiated from the black hole, where the distribution for fermions is given by

$$N_m = \frac{1}{e^{(\omega-m\Omega_H)/T_H} + 1} . \quad (18)$$

Finally, we want to find the energy-momentum flux and the Hawking temperature of the black hole. The anomalous equation of the energy-momentum tensor in the region near the event horizon is given by

$$\nabla_\mu T^\mu_\nu = F_{\mu\nu}\tilde{J}^\mu + \frac{1}{\sqrt{-g^{(2)}}}\partial_\mu N^\mu_\nu , \quad (19)$$

where  $N^\mu_\nu = \frac{1}{96\pi}\epsilon^{\beta\mu}\partial_\alpha\Gamma^\alpha_{\nu\beta}$ . By applying the same process as in [7–9], the flux of the energy-momentum is determined as

$$\begin{aligned} F_M &= \frac{m^2}{4\pi}A_t^2(r_H) + N_t^r(r_H) \\ &= \frac{m^2}{4\pi}\Omega_H^2 + \frac{1}{192\pi}f'(r_H)h'(r_H) . \end{aligned} \quad (20)$$

Comparing this result with the the energy-momentum flux derived from the Hawking distribution (18), which is given by

$$F_M = \frac{m^2}{4\pi}\Omega_H^2 + \frac{\pi}{12}T_H^2 , \quad (21)$$

one can obtain the Hawking temperature as

$$T_H = \frac{1}{4\pi} \sqrt{f'(r_H)h'(r_H)} , \quad (22)$$

which is consistent with the result calculated via the surface gravity.

Up to now, we have succeeded in applying the quantum anomaly cancellation method to derive the Hawking radiation of fermionic field from the event horizon of the 2+1 dimensional stationary black hole. This indicates that the method of anomaly cancellation to derive Hawking radiation is quite universal for different types of perturbative field.

### III. HAWKING RADIATION OF FERMIONIC FIELD IN BTZ BLACK HOLE

In this section, we consider the BTZ black hole to give a demonstration of the general discussions in the last section. The BTZ black hole solution is an exact solution to Einstein field equation in a 2 + 1 dimensional Einstein gravity with a negative cosmological constant  $\Lambda = -1/l^2$  where the action is given by

$$I = \int dx^3 \sqrt{-g} (R + 2\Lambda) . \quad (23)$$

The BTZ black hole is described by the metric[15]

$$ds^2 = -N^2 dt^2 + \frac{1}{N^2} dr^2 + r^2 (d\phi + N^\phi dt)^2 , \quad (24)$$

where

$$N^2 = -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2} , \quad N^\phi = -\frac{J}{2r^2} , \quad (25)$$

with  $M$  and  $J$  being the ADM mass and angular momentum of the BTZ black hole, respectively. The horizon is determined by the equation

$$N^2 = \frac{1}{l^2 r^2} (r^2 - r_+^2)(r^2 - r_-^2) = 0 , \quad (26)$$

where  $r_+$  and  $r_-$  are locations of outer and inner horizons respectively.  $r_+$  and  $r_-$  are given by

$$r_\pm^2 = \frac{Ml^2}{2} \left[ 1 \pm \sqrt{1 - \frac{J^2}{M^2 l^2}} \right] . \quad (27)$$

Comparing this metric with the general line element in the last section, one can easily find the near-horizon physics for the fermionic field in BTZ black hole can be approximated



by an infinite collection of two component fermionic fields in 1+1 dimensional spacetime background coupled with the dilaton field and the  $U(1)$  gauge field, which are given by

$$\begin{aligned} ds^2 &= -N^2 dt^2 + \frac{dr^2}{N^2}, \\ \Phi &= \frac{r}{2\pi}, \quad A_t = -\frac{J}{2r^2}, \quad A_r = 0. \end{aligned} \quad (28)$$

Then, from eq.(17) and (20) the angular momentum flux and energy-momentum flux are respectively given by

$$\begin{aligned} F_a &= -\frac{m^2}{2\pi} A_t(r_H) = \frac{m^2}{4\pi} \frac{J}{r_+^2}, \\ F_M &= \frac{m^2}{4\pi} A_t^2(r_H) + \frac{1}{192\pi} ((N^2)')^2(r_H) \\ &= \frac{m^2 J^2}{16\pi r_+^4} + \frac{(r_+^2 - r_-^2)^2}{48\pi l^4 r_+^2}, \end{aligned} \quad (29)$$

from which the angular velocity and Hawking temperature can be read

$$\Omega_H = \frac{J}{2r_+^2}, \quad T_H = \frac{(r_+^2 - r_-^2)}{2\pi l^2 r_+}, \quad (30)$$

which is consistent with the results in the previous literatures[16].

#### IV. HAWKING RADIATION OF FERMIONIC FIELD IN THE ROTATING BLACK HOLE IN BHT MASSIVE GRAVITY

In this section, we will consider a rotating black hole solution in BHT massive gravity theory. BHT massive gravity theory is a new theory of massive gravity recently proposed by Bergshoeff, Hohm and Townsend[17]. The theory is described by the action

$$I_{BHT} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[ R - 2\lambda - \frac{1}{m^2} \left( R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2 \right) \right], \quad (31)$$

where  $m$  is the mass parameter of the massive gravity and  $\lambda$  is a constant which is different from the cosmological constant. The field equations are then of fourth order and given by

$$G_{\mu\nu} + \lambda g_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu} = 0, \quad (32)$$

where  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$  is the Einstein tensor and

$$\begin{aligned} K_{\mu\nu} &= 2\nabla^2 R_{\mu\nu} - \frac{1}{2} (g_{\mu\nu} \nabla^2 R + \nabla_\mu \nabla_\nu R) - 8R_{\mu\alpha} R^\alpha_\nu \\ &\quad + \frac{9}{2} R R_{\mu\nu} + g_{\mu\nu} \left( 3R^{\alpha\beta} R_{\alpha\beta} - \frac{13}{8} R^2 \right). \end{aligned} \quad (33)$$

In the special case,  $m^2 = \lambda = -\frac{1}{2l^2}$ , the BHT massive gravity theory has the following rotating black hole solution [18]

$$ds^2 = -N(r)F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 (d\phi + N^\phi(r)dt)^2, \quad (34)$$

where the metric functions are given by

$$\begin{aligned} N &= \left[ 1 + \frac{bl^2}{4H} (1 - \Xi^{1/2}) \right]^2, \\ N^\phi &= -\frac{a}{2r^2} (4GM - bH), \\ F &= \frac{H^2}{r^2} \left[ \frac{H^2}{l^2} + \frac{b}{2} (1 + \Xi^{1/2}) H + \frac{b^2 l^2}{16} (1 - \Xi^{1/2})^2 - 4GM\Xi^{1/2} \right], \end{aligned} \quad (35)$$

and

$$H = \left[ r^2 - 2GMl^2 (1 - \Xi^{1/2}) - \frac{b^2 l^4}{16} (1 - \Xi^{1/2})^2 \right]^{1/2}, \quad (36)$$

with

$$\Xi = 1 - \frac{a^2}{l^2}. \quad (37)$$

The angular momentum is given by  $J = Ma$ , where  $M$  is the mass measured with respect to the zero mass black hole and  $l < a < l$  is the rotation parameter. Except for the mass  $M$  and the rotation parameter  $a$ , the solution is also described by an additional gravitational hair parameter  $b$ . The horizon is determined by the equation  $F = 0$ , which can be solved as

$$r_H = \frac{l^2}{2} \sqrt{\frac{1 + \Xi^{1/2}}{2}} \left( \sqrt{b^2 + \frac{16GM}{l^2}} - b\Xi^{1/4} \right). \quad (38)$$

Comparing the metric with the general line element in the section II, one can find the near-horizon physics for fermionic field in this black hole solution is approximated by an infinite collection of two component fermionic fields in 1+1 dimensional spacetime background coupled with the dilaton field and the  $U(1)$  gauge field, which are given by

$$\begin{aligned} ds^2 &= -NFdt^2 + \frac{dr^2}{F}, \\ \Phi &= \frac{r}{2\pi}, \quad A_t = -\frac{a}{2r^2} (4GM - bH), \quad A_r = 0. \end{aligned} \quad (39)$$

Then, from eq.(17) and (20) one can find the angular momentum flux and energy-momentum

flux which are respectively given by

$$\begin{aligned}
F_a &= -\frac{m^2}{2\pi} A_t(r_H) = \frac{m^2}{2\pi} \frac{(1 - \Xi^{1/2})}{a}, \\
F_M &= \frac{m^2}{4\pi} A_t^2(r_H) + \frac{1}{192\pi} (NF)' F'|_{r_H} \\
&= \frac{m^2}{4\pi} \frac{(1 - \Xi^{1/2})^2}{a^2} + \frac{1}{96\pi} \Xi \left( b^2 + \frac{16GM}{l^2} \right) (1 + \Xi^{1/2})^{-1},
\end{aligned} \tag{40}$$

from which the angular velocity and Hawking temperature can be read

$$\begin{aligned}
\Omega_H &= -A_t(r_H) = \frac{(1 - \Xi^{1/2})}{a}, \\
T_H &= \frac{1}{4\pi} (NF)' F'|_{r_H} = \frac{1}{2\pi} \sqrt{\frac{\Xi}{2} \left( b^2 + \frac{16GM}{l^2} \right) (1 + \Xi^{1/2})^{-1}}.
\end{aligned} \tag{41}$$

This is consistent with the results given in [18].

## V. CONCLUSION

In this paper, the method of anomaly cancellation to derive Hawking radiation initiated by Robinson and Wilczek is applied to 2+1 dimensional stationary black holes. In particular, we have found the anomaly method can be used to investigate the Hawking radiation of spin  $\frac{1}{2}$  field, which indicates the anomaly method is quite universal.

The most essential observation in this paper is that the near-horizon physics for the spin  $\frac{1}{2}$  field in the background of the general 2+1 dimensional stationary black hole can be approximated by an infinite collection of two component fermionic fields in 1+1 dimensional spacetime background coupled with the dilaton field and the  $U(1)$  gauge field. This permits us to derive Hawking radiation of fermionic field by solving the anomaly current equation for gauge field and energy-momentum tensor in the reduced two dimensional background. As an example, we also apply this method to two types of black holes in three dimensional spacetime, which are BTZ black hole in Einstein gravity and a rotating black hole in Bergshoeff-Hohm-Townsend (BHT) massive gravity. It is shown that the Hawking temperatures are recovered and they are consistent with the results obtained previously in literatures.

## ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China and Cuiying Project of Lanzhou University.

---

- [1] S.W.Hawking, Nature, 248, 30(1974).
- [2] S.W.Hawking, Commun.Math.Phys.**43**,199(1975).
- [3] J. D. Beckenstein, Phys. Rev. D **7**, 2233 (1973).
- [4] T.Damour and R.Ruffini, Phys.Rev.D **14**, 332(1976).
- [5] S. Christensen and S. Fulling, Phys.Rev.D **15**, 2088(1977).
- [6] M.K.Parikh and F.Wilczek, Phys.Rev.Lett. **85**, 5042(2000).
- [7] S. P. Robinson and F.Wilczek, Phys.Rev.Lett. **95**, 011303(2005).
- [8] S. Iso, H. Umetsu and F. Wilczek, Phys.Rev.Lett. **96**, 151302(2006).
- [9] Satoshi Iso, Hiroshi Umetsu, Frank Wilczek, Phys.Rev.D **74**,044017(2006);  
Keiju Murata, Jiro Soda, Phys.Rev. D **74**,044018(2006);  
M. R. Setare, Eur.Phys.J.C, **49**,865(2007);  
Satoshi Iso, Takeshi Morita, Hiroshi Umetsu, JHEP, **04**,068(2007);  
Qing-Quan Jiang, Shuang-Qing Wu, Phys.Lett.B **647**,200(2007);  
Qing-Quan Jiang, Shuang-Qing Wu, Xu Cai, Phys.Rev.D **75**,064029(2007);  
Kui Xiao, Wenbiao Liu, Hongbao Zhang, Phys.Lett.B **647**,482(2007);  
Hyeonjoon Shin, Wontae Kim, JHEP, **06**,012(2007);  
Qing-Quan Jiang, Shuang-Qing Wu, Xu Cai, Phys.Lett.B **651**,65(2007);  
Shuang-Qing Wu, Jun-Jin Peng, Class.Quant.Grav. **24**,5123(2007);  
Wontae Kim, Hyeonjoon Shin, JHEP, **07**,070(2007);  
Rabin Banerjee, Shailesh Kulkarni, Phys.Rev.D **77**, 024018(2008);  
Rabin Banerjee, Shailesh Kulkarni, Phys.Lett.B **659**, 827(2008);  
Rabin Banerjee, Sunandan Gangopadhyay, Shailesh Kulkarni, arXiv:0804.3492;  
Rabin Banerjee, Int.J.Mod.Phys.D **17**, 2539(2009);  
Rabin Banerjee, Shailesh Kulkarni, arXiv:0810.5683;  
Sunandan Gangopadhyay, Shailesh Kulkarni, Phys.Rev.D **77**, 024038(2008);

- Jun-Jin Peng, Shuang-Qing Wu, Phys.Lett.B **661**,300(2008);
- Xiaoning Wu, Chao-Guang Huang, Jia-Rui Sun, Phys.Rev.D **77**,124023(2008);
- Shuang-Qing Wu, Jun-Jin Peng, Zhan-Yue Zhao, Class.Quant.Grav. **25**,135001(2008);
- Sunandan Gangopadhyay, Phys.Rev.D **78**,044026(2008);
- Sunandan Gangopadhyay, Europhys.Lett.**85**,10004(2009);
- E. Papantonopoulos, P. Skamagoulis, arXiv:0812.1759;
- Shao-Wen Wei, Ran Li, Yu-Xiao Liu, Ji-Rong Ren, arXiv:0901.2614;
- Takeshi Morita, arXiv:0902.3885;
- A. P. Porfyriadis, Phys.Rev. D **79**, 084039(2009);
- A. P. Porfyriadis, Phys.Lett.B **675**, 235(2009);
- Shao-Wen Wei, Ran Li, Yu-Xiao Liu, Ji-Rong Ren, arXiv:0904.2915.
- Ran Li, Ji-Rong Ren and Shao-Wen Wei, Eur.Phys.J.C **62**,455(2009).
- Jun-Jin Peng and Shuang-Qing Wu, arXiv:0906.5121.
- [10] S. Iso, T. Morita and H. Umetsu, Phys. Rev. D **75**, 124004(2007);
- S. Iso, T. Morita and H. Umetsu, Phys. Rev. D **76**, 064015(2007);
- S. Iso, T. Morita and H. Umetsu, Phys. Rev. D **77**, 045007(2008);
- S. Iso, T. Morita and H. Umetsu, Nucl. Phys. B **799**, 60(2008);
- S. Iso, Int. J. Mod. Phys. A **23**, 2082(2008);
- L. Bonora, M. Cvitan, JHEP **0805**, 071(2008);
- L.Bonora, M.Cvitan, S.Pallua, I.Smolic, JHEP **0812**, 021(2008);
- Takeshi Morita, JHEP **0901**, 037(2009).
- [11] Subir Ghosh, arXiv:0804.2999;
- V. Akhmedova, T. Pilling, A. Gill and D. Singleton, arXiv:0808.3413;
- Rabin Banerjee, Bibhas Ranjan Majhi, Phys. Rev. D **79**, 064024(2009);
- Rabin Banerjee, Bibhas Ranjan Majhi, Phys.Lett.B **674**, 218(2009).
- [12] Keiju Murata, Umpei Miyamoto, Phys.Rev.D **76**,084038(2007).
- [13] L. Bonora, M. Cvitan, S. Pallua, and I. Smolic, Phys. Rev. D **80**, 084034(2009).
- [14] S. Nam and J.-D. Park, Class.Quant.Grav. **26**,145015(2009).
- [15] M.banados, C.Teitelboim and J.Zanelli, Phys.Rev.Lett.
- [16] R. Li, J.-R. Ren, Phys. Lett. B, **661**, 370(2008).
- [17] E. A. Bergshoeff, O. Hohm and P. K. Townsend, Phys. Rev. Lett. **102**, 201301(2009).

- [18] J. Oliva, D. Tempo and R. Troncoso, JHEP **0907**, 011(2009);  
G. Giribet1, J. Oliva, D. Tempo and R. Troncoso, arXiv: 0909.2564.